

MODEL ORDER REDUCTION AND LINEAR QUADRATIC REGULATOR CONTROLLER DESIGN, ON A LARGE SCALE LINEAR TIME INVARIANT SYSTEM

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ABSTRACT

The main objective of this paper, aims to apply model order reduction on a large scale system and design a Linear Quadratic Regulator (LQR) based controller, to analyse the performance indices, in the time and frequency domains. Control aspects of large scale systems (models with very high order) are a major concern, in the field of control systems. The order of the designed controller must be close to the order of the large scale system, or even more in most cases. As the order of the controller increases the control aspects of the system, it becomes even more complex. Evidently, there are many model order reduction techniques, that reduce the order of the higher order system, without losing the predominant characteristics. A linear quadratic regulator based design is an optimization tool, to derive an optimal controller by minimizing the cost function, based on the two weighting matrices Q and R , which weigh the state vector and the system input, respectively. The step, impulse and the frequency responses of the system with LQR controller are simulated in MatLab. In this paper, a single-input-single-output system (SISO) is considered, nevertheless due to the compatibility of LQR controller, with the state space equations, this study may be extended to multi-input-multi-output (MIMO) systems, provided the model order reduction techniques are chosen appropriately.

KEYWORDS: Higher Order System, Reduced Order Model, Linear & Quadratic Regulator

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INTRODUCTION

Complex industrial systems like power systems, chemical systems, and smart grid in recent times are large in nature and technically speaking, they are also multi input multi output systems, single or multi-objective systems and have numerous variables waiting for control. Since, the order of the system is very high, this leads to the complexities in their control. Needless to mention, these large scale systems will be controlled by a few or many controllers at a particular level in the hierarchy and the so obtained control actions may be coordinated at a higher level for further action. The controllers used in large scale systems may either operate in coordination or in a conflicting manner. Hence, the design of controllers for such higher order systems tend to become tedious, time consuming and also leads to a steep rise in the economical considerations. In order to effectively reduce these effects encountered by large scale systems, it is advisable to reduce the order of the system and hence design a lower order controller, the order of which is same as the reduced order model, provided the key properties of the higher order system are retained. One of the most important properties employed in model order reduction is to

retain the predominant characteristics of the large scale system. The step, impulse and the frequency responses of both the higher order system and the reduced order model must be matched essentially. In the step response comparison the time response specifications of the higher order and the reduced order must be spaced as closely as possible. In the frequency response matching within the specified bandwidth if the magnitude plot of the higher order system and the reduced order system matches then the behavior of the reduced model is considered to be similar to that of the higher order system. The designed controller for a reduced order system when applied on the original system should lead to a similar performance with regards to the controller designed for the original higher order system. Only then the designed lower order controller is applicable for control of original systems.

There are numerous model order reduction techniques available in the literature since the first paper published by E. J. Davison in 1968 [1]. A few of the recent methods available in literature are the pole clustering and pade approximation method [2] by Vishwakarma, the factor division method [3] and order reduction on multi variable large scale LTI systems [4] by S K Tiwari, eigen permutation and pade approximation [5] by Jay Singh, redefined time moments and markov parameters by G Saraswathi [6], [13-15] and various optimization methods [7-8]. Similarly, the design strategies employed and available in literature are the PID, LQR and LQR-PID Controller design [9] by Argentim, the comparisons of PID and LQR by Jose [10] and Nasir [11], the robust LQR controller design [12] by Kumar *et al* etc. This paper is organized in the following way: the dynamics of both the original system and the reduced order system are given in system dynamics section. The controller design is dealt with in the section LQR Controller Design. In the numerical example section an eighth order system is considered to show the effectiveness of the theory so proposed. In the next section simulated results and comparisons along with the discussions are made. Conclusions are given along with the future scope of the paper and study.

HIGHER ORDER SYSTEM DYNAMICS

The original higher order system is considered to be a linear time invariant system of n^{th} order and is represented by the following state equations

$$\dot{X} = AX + BU \quad (1)$$

$$Y = CX \quad (2)$$

Where X is the $n \times 1$ state vector, A is the $n \times n$ system matrix, B is the $n \times m$ input matrix, C is the $p \times n$ output matrix, U is the $n \times 1$ input vector and Y is the $p \times 1$ output vector, n is the number of state variables, m is the number of inputs and p is the number of output variables.

The transfer function representation of the same is given as

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \dots + b_1s + b_0}{a_ns^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1s + a_0} \quad (3)$$

Where $G(s)$ is the transfer function of the given original system, b_i ($i=0,1,2,3,\dots,n-1$) and a_i ($i=0,1,2,\dots,n$) are the scalar constants of the numerator and denominator polynomials respectively and $N(s)$ and $D(s)$ are termed as the numerator and the denominator polynomials of the given higher order transfer function.

$$D(s) = \prod_{i=0}^n (s + p_i) \quad (4)$$

Where, $p_i = (\alpha_i + j\beta_i)$, $i = 0, 1, 2, \dots, n$ are the poles of the original transfer function and

$$N(s) = \prod_{i=0}^{n-1} (s + z_i) \quad (5)$$

Where, $z_i = (\gamma_i + j\delta_i)$, $i = 0, 1, 2, \dots, n-1$ are the zeros of the original transfer function

The poles and zeros may be real and/or complex or repeated poles. If they are complex, they occur in conjugate pairs.

REDUCED ORDER SYSTEM DYNAMICS

Let us consider the reduced order system to be of k^{th} order and the system transfer function may be represented by the equation given below

$$R_k(s) = \frac{N_{k-1}(s)}{D_k(s)} = \frac{d_{k-1}s^{k-1} + d_{k-2}s^{k-2} + \dots + d_1s + d_0}{c_k s^k + c_{k-1}s^{k-1} + c_{k-2}s^{k-2} + \dots + c_1s + c_0} \quad (6)$$

Where $R_k(s)$ is the reduced k^{th} order system, d_i ($i=0,1,2,3,\dots,k-1$) and c_i ($i=0,1,2,\dots,k$) are the scalar constants of the numerator and denominator polynomials, respectively and $N_k(s)$ and $D_k(s)$ are termed as the numerator and the denominator polynomials, of the reduced k^{th} order system.

$$D_k(s) = \prod_{i=0}^k (s + rp_i) \quad (7)$$

Where, $rp_i = (ra_i + jr\beta_i)$, $i = 0,1,2,\dots,k$, are the poles of the reduced transfer function and

$$N_{k-1}(s) = \prod_{i=0}^{k-1} (s + rz_i) \quad (8)$$

Where, $rz_i = (r\gamma_i + jr\delta_i)$, $i = 0, 1, 2, \dots, k-1$, are the zeros of the reduced transfer function

LQR CONTROLLER DESIGN

An LQR controller is one of the most powerful design methodologies for systems, having complex performance requirements. It is also termed as an optimization tool, to derive an optimal controller, by minimizing the cost function, based on the two weighting matrices Q and R , which weigh the state vector and the system input respectively.

Consider the system given in equation (1), the LQR determines the gain matrix K of size $n \times m$ such that $U(t) = -KX(t)$ becomes the state feedback control law. The basic LQR control structure is depicted in figure 1 where all the states are fed back as control actions, through the optimal gain matrix determined by the LQR controller.

This feedback control law is bound to satisfy the minimal cost function $J[U(K)]$, given by

$$J[U(K)] = \frac{1}{2} \int_0^{\infty} [X^T(t)QX(t) + U^T(t)RU(t)]dt \quad (9)$$

Where Q is the non negative definite weighting matrix which controls the system states from moving out of the equilibrium and R is the positive definite weighting matrix which controls the control input.

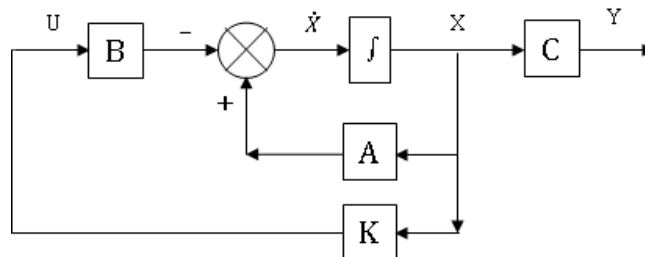


Figure 1: Basic LQR Control Structure

The following LQR design algorithm is used to determine the optimal state feedback.

Step 1: Solve the matrix Algebraic Riccati Equation (ARE)

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 \quad (10)$$

Step 2: Determine the optimal state $x^*(t)$ from

$$\dot{X}^*(t) = [A - BR^{-1}B^T P]X^*(t) \quad (11)$$

Step 3: Obtain the optimal control $u^*(t)$ from

$$U^*(t) = -R^{-1}B^T P X^*(t) \quad (12)$$

Step 4: Obtain the optimal performance index from

$$J^* = \frac{1}{2} X^{*T}(t) P X(t) \quad (13)$$

The weighting matrices Q and R play a vital role in optimizing the means to derive an optimal LQR controller. They greatly influence the performance of the system. LQR also provides liberty to the designer to freely select these matrices but technically the selection of Q and R by the designer must follow an iterative procedure who understands the intricacies/problems involved in the system thoroughly.

NUMERICAL EXAMPLE

Consider an eighth order original system

$$G(s) = \frac{N(s)}{D(s)} = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320} \quad (14)$$

The pole clustering and pade approximation method [2] by C B Vishwakarma, the factor division method [3] by S K Tiwari, eigen permutation and pade approximation [5] by Jay Singh, redefined time moments and markov parameters by G Saraswathi [6] are considered for analysis, on the determined optimal LQR controller. In this example, the original system $G(s)$ is solved in detail, to obtain the second order reduced model, by the aforementioned reduced order models, the transfer functions of which are shown in Table 1.

Using MatLab, the LQR controller's optimal gain matrix K is determined for every reduced order model, given in Table 1.

DESIGN METHODOLOGY

- The original system and the reduced order models considered are first converted into the state space models
- By selecting the suitable Q and R matrices for each system an optimal LQR controller is designed, wherein an optimal gain matrix K is obtained for each of the systems mentioned.
- The state feedback control law $u(t) = -Kx(t)$ is applied and hence, the matrix A becomes $(A-BK)$
- The step response, impulse response and the frequency response of the obtained state feedback system is measured
- If these responses are not appropriate, then the weighting matrices Q and R are tuned, to obtain the desired

performance

RESULTS AND DISCUSSIONS

Firstly, four reduced order models were used, i.e., the pole clustering and pade approximation method [2] by Vishwakarma, the factor division method [3] and order reduction on multi variable large scale LTI systems [4] by S K Tiwari, eigen permutation and Pade approximation [5] by Jay Singh, redefined time moments and Markov parameters by G Saraswathi [6]. Table 1 shows the 2nd order reduced models and the open loop step responses of all these reduced models are compared with the step response of the original eighth order system $G(s)$ in figure 2. Similarly the open loop impulse responses of all the reduced models are compared with the impulse response of the original eighth order system in figure 3. Frequency response analysis is also depicted in figure 4 for the reduced order models in comparison with the original eighth order system. These three comparisons shown in figures 2, 3 and 4 evidently show that the reduced order models follow the original system closely and match the performance.

Table 1: Reduced Order systems and Their Corresponding Controller Gains

Reduced 2 nd Order Models		Controller Gains
C B Vishwakarma <i>et al</i> [2]	$R_2(s) = \frac{N_1(s)}{D_2(s)} = \frac{16.511145s + 5.45971}{s^2 + 6.19642s + 5.45971}$	[10.4853 0.0000]
S K Tiwari <i>et al</i> [3-4]	$R_2(s) = \frac{N_1(s)}{D_2(s)} = \frac{17.74s + 5.793}{s^2 + 6.793s + 5.793}$	[10.9636 0.0000]
Jay Singh <i>et al</i> [5]	$R_2(s) = \frac{N_1(s)}{D_2(s)} = \frac{14.0097s + 4.5}{s^2 + 5.5s + 4.5}$	[8.5075 0.0000]
Saraswathi [6]	$R_2(s) = \frac{N_1(s)}{D_2(s)} = \frac{18s + 6}{s^2 + 7s + 6}$	[11.0117 0.0000]

An optimal LQR based controller, which is designed under the MatLab simulation environment is tested and applied on these reduced order models, wherein the corresponding gain matrix K is determined, by iteratively selecting the weighting matrices Q and R . The weighting matrices chosen are $Q = \begin{bmatrix} 10^{15} & 0 \\ 0 & 1 \end{bmatrix}$ and $R = [10^{12.56}]$. Thus the state feedback controller gains corresponding to reduced order models considered were determined to be $K_v = [10.4853 \ 0.0000]$ for Vishwakarma *et al*, $K_t = [10.9636 \ 0.0000]$ for Tiwari *et al*, $K_s = [8.5075 \ 0.0000]$ for Jay Singh *et al* and $K_{gs} = [11.0117 \ 0.0000]$ for Saraswathi as shown in Table 1, where K_v , K_t , K_s , K_{gs} are the state feedback gain matrices for vishwakarma, tiwari, singh and saraswathi respectively. The closed loop poles of these reduced order models, with LQR controller are $S_1 = -16.3477$ and $S_2 = -0.3340$ for Vishwakarma, $S_1 = -17.4242$ and $S_2 = -0.3325$ for Tiwari, $S_1 = -13.6785$ and $S_2 = -0.3290$ for Jay Singh and $S_1 = -17.6722$ and $S_2 = -0.3395$ for Saraswathi. The closed loop pole locations indicate that, the designed LQR controller moves the system into stable region, since all the poles lay on the left half of s -plane.

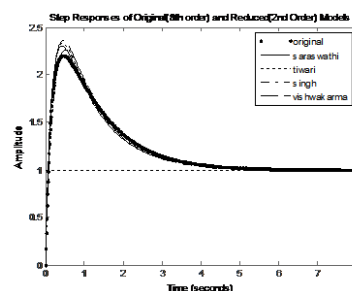


Figure 2: Step Responses of Original (8th Order) and Reduced (2nd Order) Models

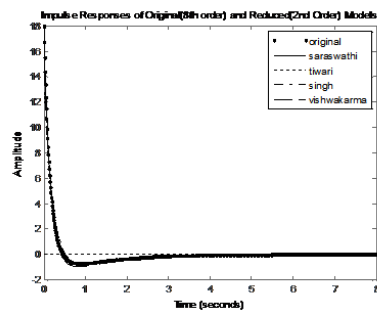


Figure 3: Impulse Responses of Original (8th Order) and Reduced (2nd Order) Models

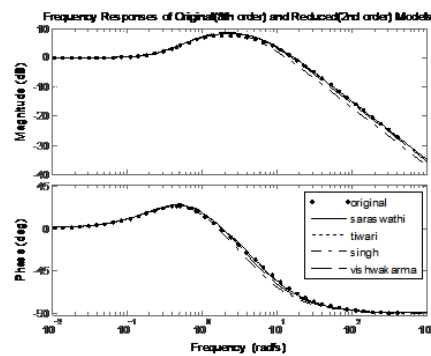


Figure 4: Frequency Responses of Original (8th Order) and Reduced (2nd Order) Models

The MatLab commands 1. $K = \text{lqr}(A, B, C, D)$ and 2. $A_c = A - B \cdot K$ calculates the state feedback gain matrix K and A_c is the system matrix of the closed loop system when the control action is fed back to the system via the state feedback gain matrix K . After application of K on the system the step, impulse and the frequency responses of all the reduced order models are represented in figures 5, 6 and 7 respectively.

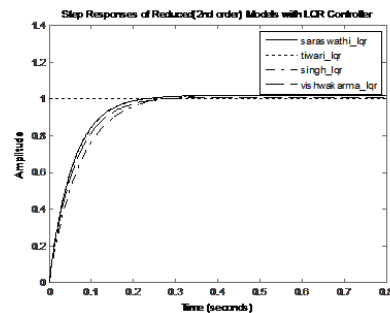


Figure 5: Step Responses of Reduced 2nd Order Models with LQR Controller

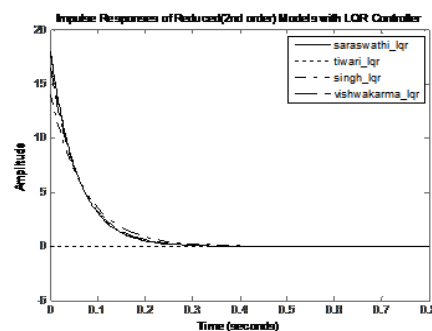


Figure 6: Impulse Responses of Reduced 2nd Order Models with LQR Controller

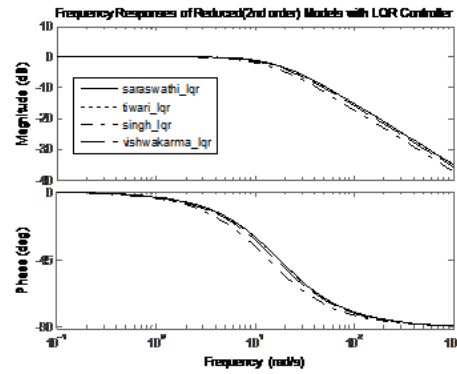


Figure 7: Frequency Responses of Reduced 2nd Order Models with LQR Controller

Dynamic performance indices such as rise time, settling time and overshoot are chosen to evaluate the performance of LQR for the step response of all the reduced order models. Based on the performance indices given in Table 2, the values of which are derived from figures 5, 6 and 7 the designed LQR controller produces similar and less rise time with a min rise time to be 0.1221 secs, for Saraswathi and a maximum rise time of 0.1548 secs for model of Jay Singh *et al.* and drives the closed loop system, to the final steady set point as quickly as 0.1867 secs and a maximum settling time of 0.626 secs. The LQR controller is also characterized by a reduced overshoot. One can easily summarize the performance of LQR controller, based on its time response specifications 1) it drives the system faster because it produces shorter settling time 2) has better robustness due to less maximum overshoot. The above few points substantiate the fact that, the LQR controller can guarantees stability and a better dynamic performance of the given system.

Table 2: Time Response Specifications of Reduced 2nd Order Models with LQR Controller

System + LQR	Rise time (secs)	Settling Time (secs)	Peak Overshoot	Steady state
Saraswathi	0.1221	0.1867	1.0159	1
Vishwakarma	0.1360	0.2164	1.0084	1
Tiwari	0.1240	0.1897	1.0155	1
Singh	0.1548	0.626	1.0202	1

Similarly, in Table 3 the frequency response specifications of reduced 2nd order models with LQR controller is provided for the reader to understand and substantiate the fact that an LQR controller which is designed in an optimal way guarantees stability and a better dynamic performance.

Table 3: Frequency Response Specifications of Reduced 2nd Order Models with LQR Controller

System + LQR	Peak Response (dB)	Phase Margin (deg)	Frequency (rad/s)	Closed Loop Stable
Saraswat	0.1295	169	3.4	Yes
Vishwak	0.0628	172	2.29	Yes
Tiwari	0.1257	169	3.32	Yes
Singh	0.1649	168	3.01	Yes

CONCLUSIONS

In this paper focus is laid on designing a Linear Quadratic Regulator (LQR) based controller and in analyzing the

performance indices of a large scale system. To design a higher order controller the design requirements becomes too complex. In order to reduce the order of the designed controller a few model order reduction techniques were used which reduced the order of the higher order system without losing the predominant characteristics. An optimal controller is derived with the help of the two weighting matrices Q and R which weight the state vector and the system input respectively. The step, impulse and the frequency responses of the system with LQR controller are simulated in Mat Lab. In this paper a single-input-single-output system is considered, due to the compatibility of LQR controller with the state space equations, this study may be extended to MIMO systems, provided the model order reduction techniques are chosen appropriately.

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